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NOTES ON THE HISTORY OF THE SLIDE RULE.*

By FLORIAN CAJORI, Colorado College, Colorado Springs.

Few instruments designed for minimizing mental labor in computation offer a more attractive field for historical study than the slide rule. Its development has reached in many directions and has attracted a great variety of intellect. Not only have writers on arithmetic been drawn to it, but also carpenters, excise officers, practical engineers, chemists, physicists, and mathematicians, including even the great Sir Isaac Newton.

And yet, the history of this instrument has been neglected to such an extent that gross inaccuracies occur in standard publications.

The first point I desire to make relates to the invention of the straight-edge slide rule. One of our American manufacturers of slide rules has an instrument on the market, called the "Gunter slide rule" and claims that it "is the original form of the slide rule." As a matter of fact Gunter never invented a slide rule. What Gunter did do was to publish in 1620, six years after Napier's publication of his logarithms, a work containing a description of Gunter's "line of numbers," which, when mounted upon a scale was called "Gunter's scale." On it distances were taken proportional to the logarithms of numbers; it was logarithms laid off upon straight lines. But Gunter's scale contained no sliding parts and, therefore, was not a slide rule.

Charles Hutton, in his *Mathematical Dictionary* (Art. "Gunter's Line"), and also in his *Mathematical Tables*, ascribes the invention of the slide rule to Edmund Wingate, 1627, but he nowhere substantiates his statement by reference to any of Wingate's works. De Morgan in his article "Slide Rule" in the *Penny Cyclopaedia* (1842), and in later publications, ascribes the invention to William Oughtred, a famous writer of mathematical text-books, 1632, and denies that Wingate ever wrote on the slide rule. It will soon appear that De Morgan was ill-informed on this subject, for he had not seen all of Wingate's works, although his criticism of a passage in Ward's *Lives of the Professors of Gresham College* (1740) is well taken. Ward claims that Wingate introduced the slide rule into France

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